was strong, in this experiment was not a little curious, the perfect image of the chess-board after bursting into view, gradually fading altogether away, and then reviving, in less strong tints, in a series of repetitions.

Another curious, though anticipated result, the author also describes under this section,—the determination, by viewing the ocular spectra, of portions of diagrams or elements in pictorial or typographical surfaces, which had not been noticed in the act of gazing. Thus, particularly on viewing a line of printed figures at a particular point, without noticing those on either side, a considerable series, right and left, were so plainly depicted on the visual organ as to be easily known; whilst, in like manner, a point in a line of a printed placard being gazed at, the lines above and below came into view on closing the eyes, and could frequently be read.

Of certain general facts elicited by this first series of investigations, the author notices, that in viewing impressions on the retina with closed eyes, all the pictures appear to occupy a position *externally*, similar to the effect when the objects are directly seen; that the spectra derived from moderate or strong degrees of transmitted light have prevalently the character of transparency, and those from very low degrees, most ordinarily, of opacity; that although many of the spectral phenomena the author had observed were well known to be capable of elicitation in the ordinary form of the experiment with the eyes open, yet the series of phenomena, as a whole, could not be so elicited, nor was it possible by such form of experiment to analyse and compare the phenomena whilst in progress of change, which, in the form he had adopted, were usually exhibited as plainly as if the spectra were the real and immediate effects of ordinary direct vision; and that such is the precision and such the certainty with which the pictures are ordinarily developed, after duly viewing any illuminated object, that the expected result, so far as the eliciting of definite pictures is concerned, hardly ever fails.

2. "On certain Properties of Square Numbers and other Quadratic Forms, with a Table by which all the odd numbers up to 9211 may be resolved into not exceeding four square numbers." By Sir Frederick Pollock, F.R.S. &c. Received Dec. 20, 1853.

In examining the properties of the triangular numbers 0, 1, 3, 6, 10, &c., the author observed that every triangular number was composed of four triangular numbers, viz. three times a triangular number plus the one above it or below it; and he found that all the natural numbers in the interval between any *two* consecutive triangular numbers might be composed of four triangular numbers having the sum of their roots, or rather of the indices of their distances from the first term of the series constant, viz. the sum of the indices of the four triangular numbers which compose the first triangular number of the *two*.

Not being at that time aware of any law by which the series that fills up the intervals could be continued, he subsequently turned his attention to the square numbers as apparently presenting a greater

variety of theorems. He observed that if any four square numbers, a^2, b^2, c^2, d^2, have their roots such, that, by making one or more positive and the rest negative, the sum of the roots may be equal to 1, then if the root or roots of which the sum is 1 less be each of them increased by 1, and the others or other be each diminished by 1, the sum of the squares of the roots thus increased or diminished will be $a^2+b^2+c^2+d^2+2$. This he found to be only a particular case of more general theorems.

Theorem A.—If the sum of the roots a, b, c, $d=2n-1$, and n be added to each of the less set, and subtracted from each of the greater, the *increase* in the sum of the squares of the new roots will be $2n$.

Theorem B.—If the sum of the roots $=2n+1$, and n be added to each of the less set and subtracted from each of the greater, the *diminution* in the sum of the squares of the new roots will be $2n$.

By means of these he shows—

Theorem C.—If any four squares be assumed which compose an odd number, these may be diminished till four squares are attained the sum of whose roots will equal 1.

By applying the first of these theorems to four roots, the sum of whose squares is an odd number, the author deduces, in a tabular form, the squares (four or less) which compose the odd numbers from 21 to 87; and remarks that there does not appear to be any limit to this mode of continuing to increase the sum of four squares by 2 each time. As, however, although this may render it probable that every odd number is composed of four, three, or two squares, it falls very short of a mathematical proof, unless it can be shown that the series can be continued by some inherent property belonging to it, he proceeds to examine the series, in order to ascertain what approach can be made to such a proof.

Adopting a method similar to that observed in the triangular numbers, the author forms what he terms the series of *Gradation*, by means of which the series of squares which compose the odd numbers may be advanced by steps or stages which increase regularly and obey a certain law, and at which this series is, as it were, commenced anew from roots of the form $n, n, n, n+1$, or $n-1, n, n, n$; the form of the sum of the squares of these roots being $4n^2\pm 2n+1$, and the series of gradation 1, 3, 7, 13, 21, 31, 43, 57, 73, &c. On this principle a more extended table of the odd numbers resolved into squares (not exceeding four in number) is constructed. On this the author remarks that it is complete to the 96th odd number (191), that is, there are in this table square numbers which will form the odd numbers in succession, whose roots (some $+$, some $-$) $=1$; and therefore the expression $4n^2\pm 2n+1$ up to $4n^2\pm 2n+191$ may be divided into 4 or 3 squares, whatever be the value of n. The numbers in the table exactly fill up the interval between

$$47^2, 47^2, 47^2, 48^2=8931,$$

and

$$47^2, 48^2, 48^2, 48^2=9121,$$

whose difference $=190$, the difference between the first term and

the last term in the table: it will therefore resolve into square numbers any odd number up to $9121+190=9211$.

With reference to the mode in which the intervals in the table may be filled up, the author states the following general theorems relating to the sums of three square numbers, by means of which the roots may be varied, and yet the sum of the squares remain the same.

Theorem D.—If any three terms of an arithmetical series, and omitting the 4th term, the three following terms be arranged thus,

$$\begin{array}{ccc} a+b, & a+2b, & a+6b, \\ a\ \ , & a+4b, & a+5b, \end{array}$$

the sum of the squares of each set of terms will be the same.

Theorem E.—If four numbers in arithmetical progression be placed thus,

$$\begin{array}{cc} a\ \ , & a+2b, \\ a+4b, & a+6b, \end{array}$$

and the sum of the 1st and 4th be divided into two parts whose difference shall be four times the arithmetic ratio, as $a+7b-(a-b)$, and the parts be placed with the terms, the greater with the less, and the less with the greater, thus,

$$\begin{array}{ccc} a\ \ , & a+2b, & a+7b, \\ a-b, & a+4b, & a+6b, \end{array}$$

the sum of the squares will be equal.

Theorem F.—Let two numbers which differ by $2n$ be placed thus:

$$\begin{array}{cc} a+n, & a+n, \\ a-n, & a-n, \end{array}$$

then if the sum of the four ($4a$) be divided so as to have the same difference ($2n$), and the parts be placed, the less with the greater, and the greater with the less, thus,

$$\begin{array}{ccc} a+n, & a+n, & 2a-n, \\ a-n, & a-n, & 2a+n, \end{array}$$

the sum of the squares shall be the same.

The author illustrates this part of the subject by deducing six forms of roots whose squares $=197$.

January 12, 1854.

The LORD CHIEF BARON, V.P., in the Chair.

Commander Kay, R.N., was admitted into the Society.

A paper was read, entitled "On some New and Simple Methods of detecting Manganese in Natural and Artificial Compounds, and